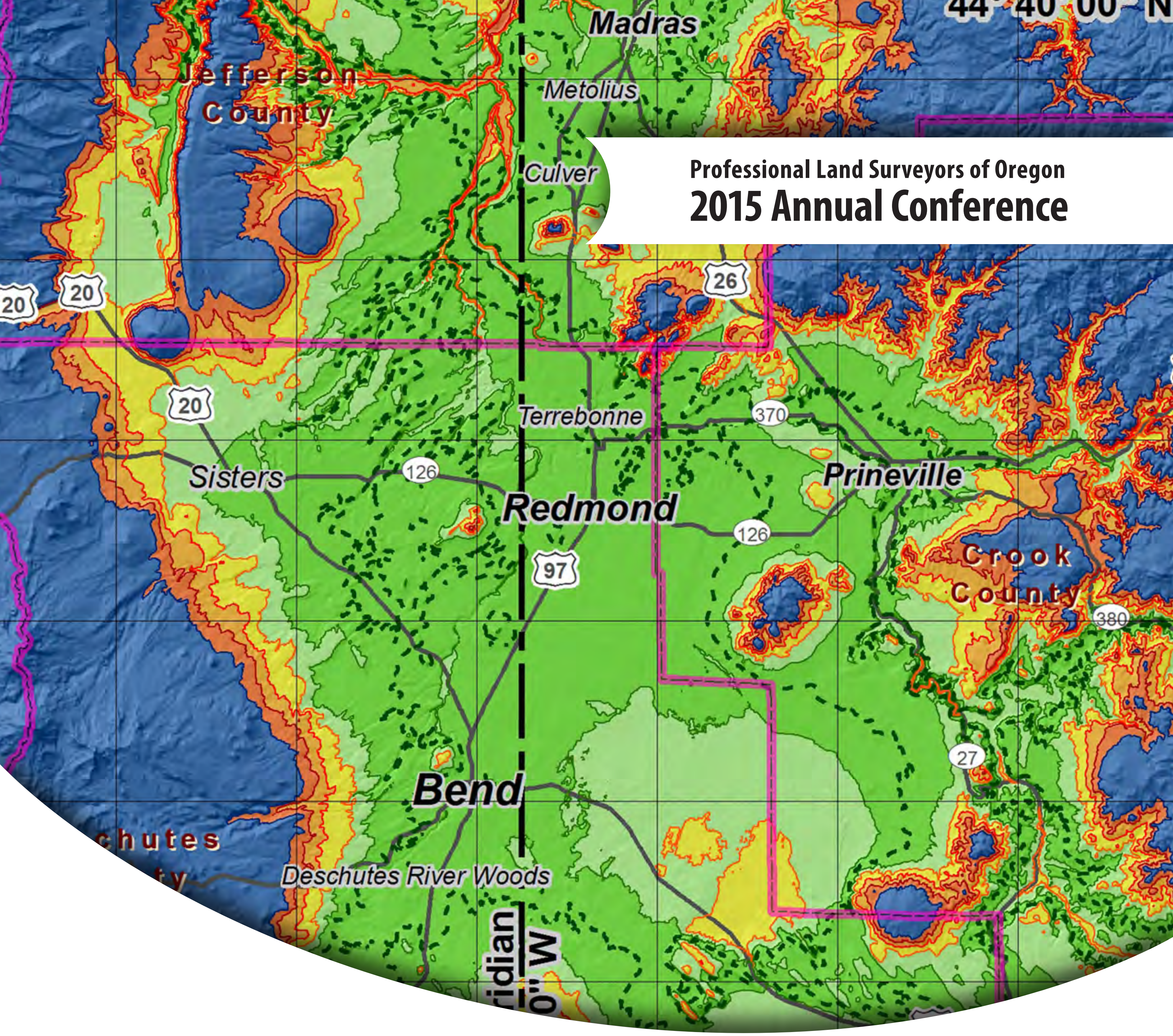
Professional Land Surveyors of Oregon
2015 Annual Conference



Ground Truth

Design and Documentation of Low Distortion Projections for Surveying and GIS

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v. 22, January 2015, Design and Metadata Example: OCRS Bend-Redmond-Prineville

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Workshop description

Projected coordinate systems are distorted — it is a Fact of Life. Linear distortion is the difference in distance between a pair of projected (map grid) coordinates and the true horizontal "ground" distance on the surface of the Earth. This difference can be significant for existing published coordinate systems, such as State Plane (e.g., up to 1 foot per mile in Bend, Oregon), and it can lead to confusion about which distances are "correct." Although such distortion cannot be eliminated, it can be minimized using Low Distortion Projections (LDPs). LDPs are conformal map projections intended to cover the largest area with the least linear distortion possible. These goals are at odds with one another, so LDP development is an optimization problem that can be complex. This workshop presents a method for designing LDPs that are fully compatible with both survey and GIS data, and yet are rigorously georeferenced. Such systems can be used directly to represent conditions "at ground" for a variety of geospatial products and services, such as survey plats, engineering plans, as-built surveys, construction staking, and legal boundary descriptions. The workshop also includes comparison to other methods of creating "ground" coordinates, examples of existing LDP systems (such as the Oregon Coordinate Reference System), and development of documentation (metadata). LDPs provide a means for survey and GIS data to coexist without using poorly defined "ground" systems or resorting to approximate "rubber-sheeting" acts of desperation.

Speaker biography

Michael L. Dennis, RLS, PE is owner of Geodetic Analysis, LLC. His firm provides geodetic consulting and surveying services, including coordinate system design, control and boundary surveys, GPS baseline processing, planning and execution of National Geodetic Survey (NGS) "Bluebook" surveys (including Height Modernization surveys), spatial data management, survey and GIS data integration, development of field and office procedures for surveying and mapping, providing educational seminars throughout the US, and creation of custom computer algorithms. Mr. Dennis is on the board of the American Association for Geodetic Surveying (AAGS) and is Chair of the AAGS Geodetic Education and Certification Committee. He is a member of the Arizona Professional Land Surveyors Association, the American Society of Civil Engineers - Geomatics Division, and the American Society for Photogrammetry and Remote Sensing. Mr. Dennis is also a geodesist at NGS and is currently on long-term leave of absence while pursuing a PhD in Geomatics Engineering and GIS at Oregon State University.

What is map projection distortion?

Map projection distortion is an *unavoidable* consequence of attempting to represent a curved surface on a flat surface. It can be thought of as a change in the "true" relationship between points located on the surface of the Earth and the *representation* of their relationship on a plane. Distortion cannot be eliminated — it is a ***F***act of ***L***ife. The best we can do is to *minimize* the effect.

There are two general types of map projection distortion:

1. Linear distortion. Difference in distance between a pair of projected (map grid) coordinates when compared to the true horizontal ("ground") distance, denoted here by $\boldsymbol{\delta}$.
    - Can express as a ratio of distortion length to ground length:
        - E.g., feet of distortion per mile; parts per million (= mm per km).
        - *Note:* 1 foot / mile = 189 ppm = 189 mm / km.
    - Linear distortion can be positive or negative:
        - NEGATIVE distortion means the projected (map grid) length is SHORTER than the "true" horizontal (ground) length.
        - POSITIVE distortion means the projected (map grid) length is LONGER than the "true" horizontal (ground) length.
    - Minimizing linear distortion ***only*** makes sense for ***conformal*** projections.
        - For conformal projections, linear distortion (scale error) is the same in every direction from a point. There are many types of conformal projections, although only a few are of practical use for Low Distortion Projections (LDPs). Conformal projections that are not practical include those for the entire Earth and ones that are not widely available or are difficult to implement or add to commercial software. The following five types are well known, although the last two are typically not optimal for LDPs.
            1. Transverse Mercator (TM). Widely used for large-scale mapping; also called the Gauss-Krüger projection.
            2. Lambert Conformal Conic (LCC). Includes both the one-parallel and two-parallel version, although the two are mathematically identical. It is also widely used for large-scale mapping, but some software is limited to the two-parallel version without the ability to scale, which thus cannot be used for LDPs (but this limitation is becoming less common in commercial software).
            3. Oblique Mercator (OM). Not used as often as the TM and LCC projections, but becoming more widely available in commercial software. A common implementation is known as the Hotine OM (also called Rectified Skew Orthomorphic).

4. Stereographic. Includes oblique and oblique aspects; also known as Double Stereographic projection. Suitable for LDPs, especially in small areas, but performance in such cases is no better than TM or LCC. For large areas, it has the disadvantage of not conforming to Earth curvature in any direction. So performance is almost always inferior to TM, LCC, or OM, and LDP design based in this projection are not covered in this document. But it may be a good choice for LDPs in polar regions or other special cases (e.g., bowl-shaped topographic areas).

5. Regular Mercator. Not suitable for most large-scale mapping, due to extreme distortion and rapid change in distortion in non-equatorial regions. But could perform well for some LDP designs near the equator.

   There are many others conformal projections. One that may be of interest for minimizing distortion on long linear projects (such as highways and railroads) is a proprietary solution called "SnakeGrid" (see www.snakegrid.org for more information). Others are typically not suitable for LDPs, including Bipolar Oblique Conic Conformal, Modified-Stereographic Conformal, Space Oblique Mercator, Peirce Quincuncial, Guyou, Adams, Littrow, etc.

- For ***all*** non-conformal projections (such as equal area projections), linear distortion generally varies with direction, so there is no single unique linear distortion (or scale error) at any point.

2. Angular distortion. For conformal projections, this equals the *convergence* (*mapping*) *angle*, γ. The convergence angle is the difference between grid (map) north and true (geodetic) north – a very useful property for surveying and engineering applications.

- Convergence angle is zero on the projection central meridian, positive east of the central meridian, and negative west of the central meridian.
- Magnitude of the convergence angle increases with distance from the central meridian, and its rate of change increases with increasing latitude, as shown in Table 1.
- For the Oblique Mercator projection, there is no central meridian (i.e., longitude along which the convergence angle is zero). However, the meridian passing through the local origin has a convergence very close to zero, and the values in this table can be used for the convergence angle one mile east or west of the Oblique Mercator local origin.
- Usually convergence is not as much of a concern as linear distortion, and it can only be minimized by staying close to the projection central meridian (or limiting surveying and mapping activities to equatorial regions of the Earth). Note that the convergence angle is zero for the regular Mercator projection, but this projection is not suitable for large-scale mapping in non-equatorial regions.

Table 1. Convergence angles at a distance of one mile (1.6 km) east (positive) and west (negative) of central meridian for Transverse Mercator projection (and Lambert Conformal Conic projection with central parallel equal to latitude in table).

Latitude	Convergence 1 mi from CM	Latitude	Convergence 1 mi from CM	Latitude	Convergence 1 mi from CM
0°	0° 00′ 00″	30°	±0° 00′ 30″	60°	±0° 01′ 30″
5°	±0° 00′ 05″	35°	±0° 00′ 36″	65°	±0° 01′ 51″
10°	±0° 00′ 09″	40°	±0° 00′ 44″	70°	±0° 02′ 23″
15°	±0° 00′ 14″	45°	±0° 00′ 52″	75°	±0° 03′ 14″
20°	±0° 00′ 19″	50°	±0° 01′ 02″	80°	±0° 04′ 54″
25°	±0° 00′ 24″	55°	±0° 01′ 14″	85°	±0° 09′ 53″

One can think of linear distortion as the unavoidable consequence of the projection “developable surface” (plane, cone, or cylinder) departing from the reference ellipsoid. Although no ellipsoidal forms of conformal projections are “perspective” (i.e., cannot be created geometrically), it is still useful to think of linear distortion increasing as the “distance” of the developable surface from the ellipsoid increases. In that sense, linear distortion is entirely a function of “height” with respect to the ellipsoid.

Yet it is convenient to consider total linear distortion as a combination of distortion due to Earth curvature and distortion due to ground height above the reference ellipsoid. Indeed, this “total” distortion is often computed as the product of these two types and is called the “combined” scale error (or factor). The relative magnitude of each component of distortion depends on the variation in topographic height and the size of the projected area.

Diagrams on the following pages illustrate distortion as geometric departure of the developable surface from the reference ellipsoid. Although the following tables give distortion values for their respective “curvature” and “height” components, total distortion is always a combination of both.

A note on terminology: The “projection axis” is the line along which projection scale (i.e., distortion with respect to the ellipsoid) is *minimum* and *constant*. It is the central meridian for the Transverse Mercator (TM) projection, the central parallel for the Lambert Conformal Conic (LCC) projection, and the skew axis for the Oblique Mercator (OM) projection (actually the scale is not quite constant along the skew axis but changes slowly with distance from OM local origin). The Stereographic projection (both oblique and polar aspects) does not have a projection axis *per se*; its scale is instead defined as a minimum at its origin point. For the regular Mercator, the projection axis is the equator.

Table 2. Horizontal linear distortion of projected coordinates due to Earth curvature.

Maximum zone width for secant projections (km and miles)	Maximum linear horizontal distortion, δ		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
16 miles (26 km)	±1 ppm	±0.005 ft/mile	1 : 1,000,000
35 miles (56 km)	±5 ppm	±0.026 ft/mile	1 : 200,000
50 miles (80 km)	±10 ppm	±0.05 ft/mile	1 : 100,000
71 miles (114 km)	±20 ppm	±0.1 ft/mile	1 : 50,000
112 miles (180 km)	±50 ppm	±0.3 ft/mile	1 : 20,000
158 miles (254 km) e.g., SPCS*	±100 ppm	±0.5 ft/mile	1 : 10,000
317 miles (510 km) e.g., UTM†	±400 ppm	±2.1 ft/mile	1 : 2,500

*State Plane Coordinate System; zone width shown is valid between ~0° and 45° latitude
†Universal Transverse Mercator; zone width shown is valid between ~30° and 60° latitude

Rule of thumb for ±5 ppm distortion (due to curvature):

Distortion due to curvature is within ±5 ppm for an area 35 miles wide (perpendicular to projection axis).

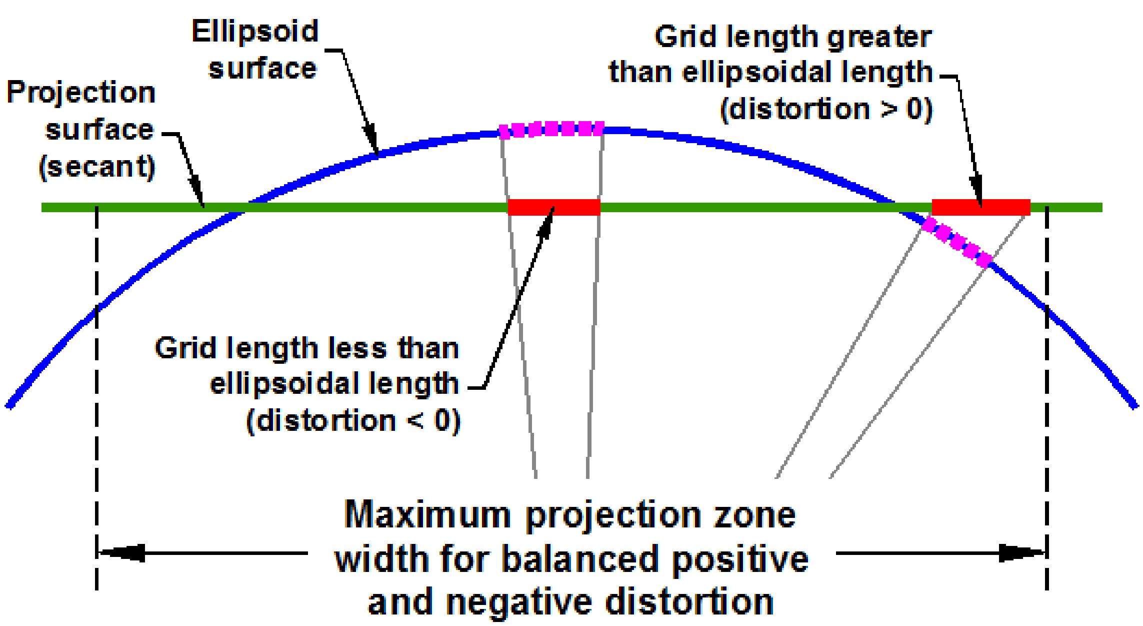

Figure 1. Sketch of linear distortion of projected coordinates due to Earth curvature.

Table 3. Horizontal linear distortion of projected coordinates due to ground height above the reference ellipsoid.

Height below (–) and above (+) projection surface	Maximum linear horizontal distortion, δ		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
±100 ft (±30 m)	±4.8 ppm	±0.025 ft/mile	~1 : 209,000
±400 ft (±120 m)	±19 ppm	±0.10 ft/mile	~1 : 52,000
±1000 ft (±300 m)	±48 ppm	±0.25 ft/mile	~1 : 21,000
+2000 ft (+600 m)*	–96 ppm	–0.50 ft/mile	~1 : 10,500
+3300 ft (+1000 m)**	–158 ppm	–0.83 ft/mile	~1 : 6,300
+14,400 ft (+4400 m)†	–688 ppm	–3.6 ft/mile	~1 : 1,500

*Approximate mean topographic height of North America (US, Canada, and Central America)
** Approximate mean topographic height of western coterminous US (west of 100°W longitude)
† Approximate maximum topographic height in coterminous US

Rule of thumb for ±5 ppm distortion (due to change in topographic height):
Distortion due to change in topographic height is within ±5 ppm for a ±100 ft range in height.

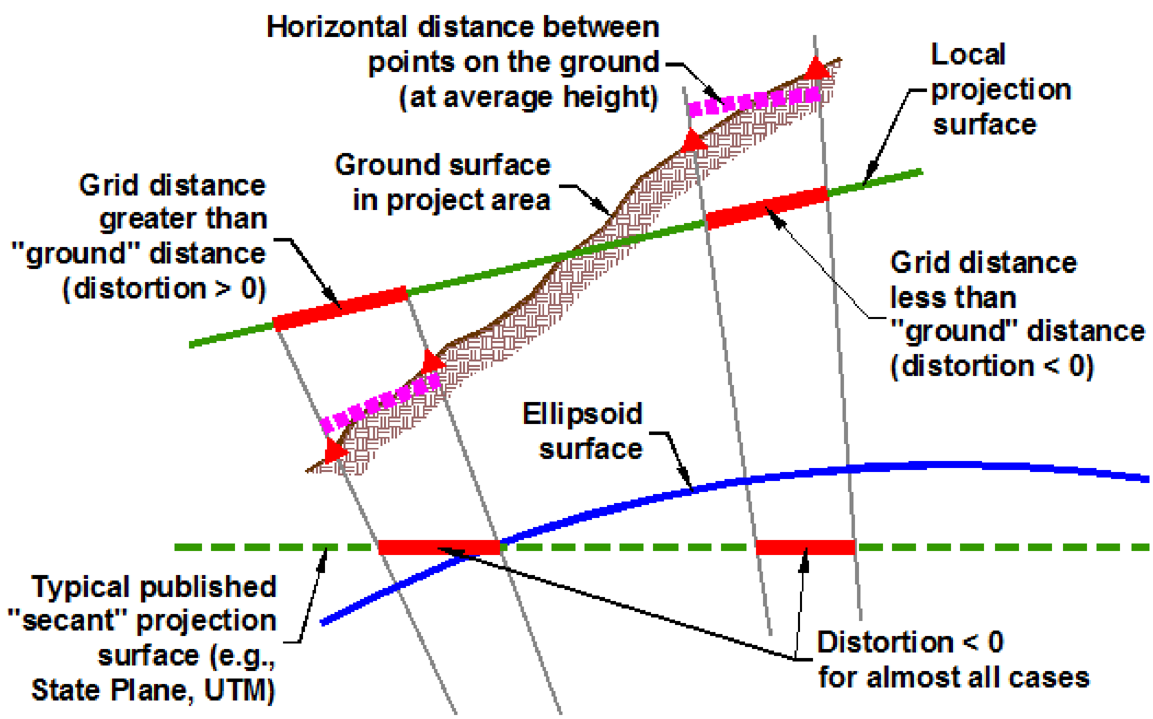

Figure 2. Sketch of linear distortion due to ground height above the reference ellipsoid.

Methods for creating low-distortion grid coordinate systems

1. **Design a Low Distortion Projection (LDP) for a specific project geographic area**

   Use a conformal projection referenced to the existing geodetic datum (d*escribed in detail later in this document*).

2. **Scale the reference ellipsoid "to ground"**

   A map projection referenced to this new "datum" is then designed for the project area.

   Problems:

   - Requires a new ellipsoid (datum) for every coordinate system, which makes it more difficult to implement than an LDP.
   - New datum makes it more complex than an LDP, yet it does not perform any better.
   - *Generates new set of latitudes that can be substantially different from original latitudes.*
     - Change in latitude can exceed 3 feet per 1000 ft of topographic height, depending on method used for scaling the ellipsoid (this case is for scaling with constant flattening).
     - Can lead to confusion over which latitude values are correct.

3. **Scale an existing published map projection "to ground"**

   Often referred to as "modified" State Plane when an SPCS projection definition is scaled.

   Problems:

   - Generates coordinates with values similar to "true" State Plane (can cause confusion).
     - Can eliminate this problem by translating grid coordinates to get smaller values.
   - Often yields "messy" parameters when a projection definition is back-calculated from the scaled coordinates (in order to import the data into a GIS).
     - More difficult to implement in a GIS, and may cause problems due to rounding or truncating of "messy" projection parameters (especially for large coordinate values).
     - Can reduce this problem through judicious selection of "scaling" parameters.
   - Does ***not*** reduce the convergence angle (it is same as that of original SPCS definition).
     - In addition, the *arc-to-chord correction* may be significant; it can reach ½ arc-second for a 1-mile line located 75 miles from the projection axis (this correction is used along with the convergence angle for converting grid azimuths to geodetic azimuths).
   - ***MOST IMPORTANT:*** **Usually does not minimize distortion over as large an area as the other two methods**
     - Extent of low-distortion coverage generally *decreases* as distance *increases* from projection axis (i.e., central meridian for TM and central parallel for LCC projection).
     - State Plane axis usually does NOT pass through the project area and may be oriented perpendicular to the long dimension of a project, decreasing area of coverage.
     - Figure 3 illustrates this problem with "modified" SPCS as compared to LDPs.

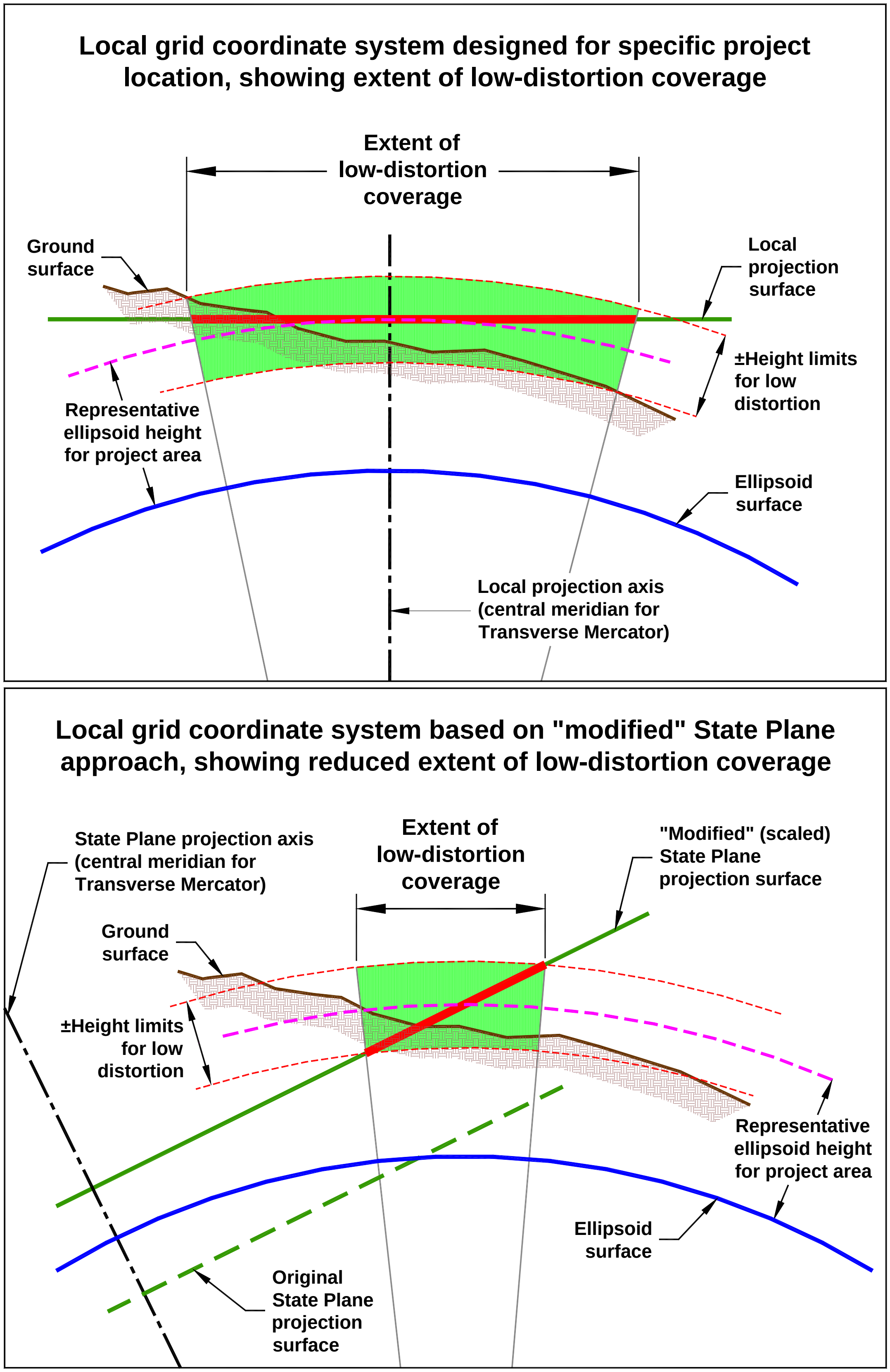

Figure 3. Comparison of distortion between LDP and typical “modified” State Plane system.

Six steps for designing a Low Distortion Projection (LDP)

The design objective is usually to minimize linear distortion over the largest area possible. These goals are at odds with one another, so LDP design is an ***optimization problem***. The following six steps are intended to address commonly encountered situations. However, often the most important part is not technical – getting concurrence among the many stakeholders impacted by the design, such as surveyors, engineers, GIS professionals, and both public and private organizations that make use of geospatial data in the design area.

1. Define distortion objective for area of interest and determine *representative* ellipsoid height, h_0 (<u>not</u> elevation)

NOTE: *This is just to get the design process started.* Ellipsoid height by itself is unlikely to yield the final design scale, except for small areas, due to curvature and/or systematic change in topographic height.

- A common objective for "low distortion" is ±20 ppm (±0.1 ft/mile), but this may not be achievable due to range of topographic height and/or size of design area. The following "rules of thumb" (from pages 5 and 6) can help guide the initial design. However, often it is possible to achieve better results than these guidelines indicate, because both height and areal extent affect distortion simultaneously, and one can be used to compensate for the other.
    - <u>Size of design area</u>. **Distortion due to curvature is within ±5 ppm for an area 35 miles wide**. Note that this width is perpendicular to the projection axis (e.g., east-west for TM and north-south for LCC projections. The effect is not linear; range of distortion due to curvature increases rapidly with increasing zone width and is proportional to the square of the zone width, i.e., doubling the zone width increases the distortion by about a factor of four (for this case, doubling zone width to 70 miles quadruples the distortion range to about ±20 ppm).
    - <u>Range in topographic ellipsoid height</u>. **Distortion due to change in topographic height is within about ±5 ppm for a ±100 ft range in height.** Note that this is essentially linear, i.e., a range of ±400 ft in height corresponds to a range of about ±20 ppm distortion.
- The *average* height of an area may not be appropriate (e.g., for projects near mountains).
    - There is no need to estimate height to an accuracy of better than about ±20 ft (±6 m); this corresponds to about 1 ppm distortion. In addition, the initial projection scale determined using this height will likely be refined later in the design process.

2. Choose projection type and place projection axis near centroid of project area

NOTE: *This is just to get the design process started.* In cases where the topography generally changes in one direction, offsetting the projection axis can yield substantially better results.

- Select a well-known and widely used ***conformal*** projection, such as the Transverse Mercator (TM), Lambert Conformal Conic (LCC), or Oblique Mercator (OM).

- When minimizing distortion, it will not always be obvious which projection type to use, but for small areas (< ~35 miles or ~55 km wide perpendicular to the projection axis), usually both the TM and LCC will provide satisfactory results. However, significantly better performance can be obtained in many cases when a projection is used with its axis perpendicular to the general topographic slope of the design area (more on this below).
- In nearly all cases, a two-parallel LCC should ***not*** be used for an LDP with the NAD 83 datum definition (but note that some software may not support a one-parallel LCC). A two-parallel LCC should not be used because the reason there are two parallels is to make the projection secant to the ellipsoid (i.e., the central parallel scale is less than 1). This is at odds with the usual objective of scaling the projection so that the developable surface is at the topographic surface, which is typically above the ellipsoid, particularly in areas where reduction in distortion is desired.
- The OM projection can be very useful for minimizing distortion over large areas, especially areas that are more than about 35 miles (55 km) long in an oblique direction. It can also be useful in areas where the topographic slope varies gradually and more-or-less uniformly in a direction other than north-south or east-west. The disadvantage of this projection is that it is more difficult to evaluate, since another parameter must be optimized (the projection skew axis). In addition, this projection is more complex, and may not be available in as many software packages as the TM and LCC projections.
- When choosing a projection, bear in mind that universal commercial software support, although desirable, is not an essential requirement for selecting a projection. In the rare cases where third parties must use a coordinate system based on a projection not supported in their software, it is always possible for them to get on the coordinate system implicitly (for example by using a best-fit procedure based on coordinate at common points).

- Placing the projection axis near the design area centroid is often a good first step in the design process (or, for the OM projection, parallel to the long axis of the design area).
  - In cases where topographic height increases more-or-less uniformly in one direction, dramatically better performance can be achieved by offsetting the projection axis from the project centroid. In such cases a projection type should be chosen such that its projection axis is perpendicular to the topographic slope (e.g., for topography sloping east-west, a TM projection should be used; for slope north-south an LCC projection should be used. The axis is located such that the developable surface best coincides with the topographic surface.
  - Often the central meridian of the projection is placed near the east-west “middle” of the project area in order to minimize convergence angles (i.e., the difference between geodetic and grid north). The central meridian is the projection axis only for the TM projection; its location has no affect on distortion for the LCC projection.

3. Scale projection axis to representative ground height, h_0

NOTE: *This is just to get the design process started.* Ellipsoid height by itself is unlikely to yield the final design scale, except for small areas, due to curvature and/or systematic change in topographic height.

- Compute map projection axis scale factor "at ground": $k_0 = 1 + \frac{h_0}{R_G}$
  - For TM projection, k_0 is the central meridian scale factor.
  - For one-parallel LCC projection, k_0 is the standard (central) parallel scale factor.
  - For OM projection, k_0 is the scale at the local origin.
- R_G is the geometric mean radius of curvature, $R_G = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$

  and φ = geodetic latitude of point, and for the GRS-80 ellipsoid:

  a = semi-major axis = 6,378,137 m (exact) = 20,925,646.325 international ft
  = 20,925,604.474 US survey ft

  e^2 = first eccentricity squared = $2f - f^2$

  f = geometric flattening = 1 / 298.257222101
  - Alternatively, can initially approximate R_G since k_0 will likely be refined in Step #4:

Table 4. Geometric mean radius of curvature at various latitudes for the GRS-80 ellipsoid (rounded to nearest 1000 meters and feet).

Latitude	R_G (meters)	R_G (feet)	Latitude	R_G (meters)	R_G (feet)
0°	6,357,000	20,855,000	50°	6,382,000	20,938,000
10°	6,358,000	20,860,000	60°	6,389,000	20,961,000
20°	6,362,000	20,872,000	70°	6,395,000	20,980,000
30°	6,367,000	20,890,000	80°	6,398,000	20,992,000
40°	6,374,000	20,913,000	90°	6,400,000	20,996,000

4. Compute distortion throughout project area and refine design parameters

- Distortion computed at a point (at ellipsoid height *h*) as $\delta = k\left(\frac{R_G}{R_G + h}\right) - 1$
    - Where k = projection grid point scale factor (i.e., distortion with respect to the ellipsoid at a point). Note that computation of k is rather involved, and is often done by commercially available software. However, if your software does not compute k, or if you want to check the accuracy of k computed by your software, equations for doing so for the TM and LCC projections are provided later in this document.
    - Multiply δ by 1,000,000 to get distortion in parts per million (ppm).
- Best approach is to compute distortion over entire area and generate a distortion map (this ensures optimal low-distortion coverage).
    - Often requires repeated evaluation using different k_0 values.
    - May warrant trying different projection axis locations and different projection types.
- General approach for computational refinement:
    - Compute distortion statistics, such as mean, range, and standard deviation.
    - Changing the projection scale only affects the mean distortion; it has essentially no affect on the variability (standard deviation and range).
    - The only way to reduce distortion variability is by moving the projection axis and/or changing the projection type. The usual objective is to minimize the distortion standard deviation and range. Once this is done, the scale can be changed so that the mean distortion is near zero.
    - Finally, check to ensure the desired distortion is achieved in important areas, and also check to ensure overall performance is satisfactory (using a map showing distortion everywhere).

5. Keep the definition SIMPLE and CLEAN!

- Define k_0 to no more than SIX decimal places, e.g., 1.000175 (exact).
    - *Note:* A change of one unit in the sixth decimal place (±1 ppm) equals distortion caused by a 20 ft (6 m) change in height.
    - For large areas with variable relief, scale defined to five decimal places (±10 ppm) is often sufficient.
- Define the central meridian and latitude of grid origin to nearest whole arc-minute; for moderate to large areas they can often be defined to the nearest five arc minutes (e.g., central meridian = 121°15’00” W).
- Define grid origin using whole values with as few digits as possible (e.g., false easting = 50,000 for a system with maximum easting coordinate value < 100,000). Note that the grid origin definition has no affect whatsoever on the map projection distortion.

  - It is strongly recommended that the coordinate values everywhere in the design area be distinct from other coordinate system values for that area (such as State Plane or UTM) in order to reduce the risk of confusing the LDP with other systems. For multi-zone LDPs, it could similarly be helpful to keep coordinates between the zones distinct, if possible.
  - Often it is desirable to define grid origins such that the northings and eastings do not equal one another anywhere in the design area.
  - In some applications, there may be an advantage to using other criteria for defining the grid origin. For example, it may be desirable for all coordinates in the design area to have the same number of digits (such as six digits, i.e., between 100,000 and 999,999). In other cases it may be useful to make the coordinates distinct from State Plane by using larger rather than smaller coordinates, especially if the LDP covers a very large area. In multi-zone systems, it may also be helpful to define grid origins such that the values correlate to zone numbers (e.g., a false easting of 3,000,000 m for zone #3).

6. *Explicitly* define linear unit and geometric reference system (i.e., geodetic datum)

- Linear unit, e.g., meter (*or* international foot, *or* US survey foot, or…?)
  - The international foot is shorter than the US survey foot by 2 ppm. Because coordinate systems typically use large values, it is critical that the type of foot used be identified (the values differ by 1 foot per 500,000 feet).
  - Because of the possibility of confusion between the international and US survey foot, it is recommended that the design parameters for the LDP be in meters (this approach is used in most State Plane zones). Output coordinates can then be specified for which type of foot is desired. It can be difficult to detect an implementation that used the incorrect type of foot, since they differ by only 2 ppm.
- Geometric reference system (geodetic datum), e.g., North American Datum of 1983 (NAD 83)
  - The reference system realization (i.e., "datum tag") should not be included in the coordinate system definition (just as it is not included in State Plane definitions). However, the datum tag *is* an essential component for defining the spatial data used within the coordinate system (as shown in a metadata example later in this document). For NAD 83, the NGS convention is to give the datum tag in parentheses after the datum name, usually as the year in which the datum was "realized" as part of a network adjustment. Common datum tags for NGS horizontal control are listed below:
    - "2011" for the current NAD 83 (2011) epoch 2010.00 realization, which is referenced to the North America tectonic plate.
    - "2007" for the (superseded) NSRS2007 (National Spatial Reference System of 2007) realization. Functionally equivalent to the superseded "CORS" datum tag and referenced to an epoch date of 2002.00 for most of the coterminous US.

    - "199*x*" for the various supersede HARN (or HPGN) realizations, where *x* is the last digit of the year of the adjustment (usually done for a particular state. The Iowa HARN is 1996 (HARN is "High Accuracy Reference Network" and HPGN is "High Precision Geodetic Network").

- *Note regarding the vertical component of a coordinate system definition:* The vertical reference system (datum) is an essential part of a three-dimensional coordinate system definition. But LDPs are restricted exclusively to horizontal coordinates. So although the vertical component is essential for most applications, it is not part of an LDP and must be defined separately. Specifically, it should be specified as part of the overall coordinate system metadata (as shown in the metadata example later in this document). A complete three-dimensional coordinate system definition must include a vertical "height" component. Typically the vertical part consists of ellipsoid heights relative to NAD 83 (when using GNSS) and/or orthometric heights ("elevations") relative to the North American Vertical Datum of 1988 (NAVD 88). These two types of heights are related (at least in part) by a hybrid geoid model, such as GEOID12A, as well as some sort of vertical adjustment or transformation to match local vertical control for a project. The approach used for the vertical component usually varies from project to project and requires professional judgment to ensure it is defined correctly. Providing such instructions is beyond the scope of this document.

- *Note regarding the relationship between NAD 83 and WGS 84:* For the purposes of entering the LDP projection parameters into particular vendor software, the datum should be defined as NAD 83 (which uses the GRS-80 reference ellipsoid for all realizations). Some commercial software implementations assume there is no transformation between WGS 84 and NAD 83 (i.e., all transformation parameters are zero). Other implementations use a non-zero transformation, and in some cases both types are available in a single software package. The type of transformation used will depend on specific circumstances, although often the zero transformation is the appropriate choice (even though it is not technically correct). Check with software support to ensure the appropriate transformation is being used for your application. Additional information about WGS 84 is available from the National Geospatial-Intelligence Agency (2014a).

Design example for a Low Distortion Projection (LDP)

Design a Low Distortion Projection (LDP) for the **Bend-Redmond-Prineville region** of Oregon.

1. **Define distortion objective for area of interest and determine *representative* ellipsoid height, h_0 (not elevation)**

   The overall design objective is ±20 ppm for the region and ±10 ppm within the three largest cities (Bend, Redmond, and Prineville). Four additional towns – Madras, Sisters, Culver, and Metolius – are also used to define the overall design region. To get the process started, ellipsoid heights were obtained at arbitrary locations in each of the seven towns using NAVD 88 orthometric heights (elevations) from the 1/3 arc-second USGS National Elevation Dataset with GEOID12A hybrid geoid heights. These values are given in Table 5, which gives a mean topographic ellipsoid height of h_0 **= 2800 ft** (which we will take as "representative" for initial design purposes)

Table 5. The seven locations (towns) in the project region used to perform LDP design.

Location	NAD 83 latitude	NAD 83 longitude	Topographic height at location (feet)		
			NAVD 88 orthometric	GEOID12A hybrid geoid	NAD 83 ellipsoid
Bend	44° 03' 29" N	121° 18' 55" W	3625	-68.8	3557
Redmond	44° 16' 21" N	121° 10' 26" W	3000	-69.5	2931
Prineville	44° 17' 59" N	120° 50' 04" W	2880	-67.5	2813
Madras	44° 38' 00" N	121° 07' 46" W	2242	-70.0	2172
Sisters	44° 17' 27" N	121° 32' 57" W	3186	-70.1	3116
Culver	44° 31' 32" N	121° 12' 47" W	2631	-69.8	2561
Metolius	44° 35' 11" N	121° 10' 42" W	2537	-70.1	2467
		Mean	**2872**	**-69.4**	**2802**
		Range	**1383**	**2.6**	**1385**
		Std deviation	**±456**	**±1.0**	**±457**

- Size of design area. The overall design area is about 40 miles long north-south, and about 35 miles wide east-west. Based on the rule of thumb of ±5 ppm distortion for a zone width of 35 miles, it appears the design distortion can be achieved, at least with respect to Earth curvature.
- Range in topographic ellipsoid height. The height range in Table 5 is 1385 ft (i.e., ±692 ft), which corresponds to about ±35 ppm based on the ±5 ppm per ±100 ft rule of thumb – not an encouraging observation, considering the design objectives of ±20 ppm overall and especially of ±10 ppm in Bend, Redmond, and Prineville.

2. Choose projection type and place projection axis near centroid of project area

Upon initial inspection, it's not clear which projection type would be best, so will evaluate both TM and LCC. To get the process started, the projection axes placed near the center of the region.

For the TM projection, the initial central meridian is set at $\boldsymbol{\lambda_0}$ **=121° 15' 00" W**.

For the LCC projection, the initial central parallel is set at $\boldsymbol{\varphi_0}$ **=44° 20' 00" N**.

Because the design area is somewhat longer north-south than east-west (40 vs. 35 miles), the TM projection may be the better choice. On the other hand, the topographic height overall decreases from north to south, which tends to favor the LCC projection. The performance of these projections will be evaluated as part of the design process.

3. Scale central meridian of projection to representative ground height, $\boldsymbol{h_0}$

First compute Earth radius at mid-latitude of φ = <u>44° 20' 00" N</u> (same as central parallel for initial LCC design):

$$R_G = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi} = \frac{20{,}925{,}646.325\times\sqrt{1-0.006694380023}}{1-0.006694380023\times\left[\sin(44.333333°)\right]^2} = \textbf{20,923,900 ift}$$

Thus the central meridian scale factor scaled to the representative ellipsoid height is

$$k_0 = 1+\frac{h_0}{R_G} = 1+\frac{2800}{20{,}923{,}900} = \textbf{1.00013} \text{ (to five decimal places)}$$

Based on these results, the following initial TM and LCC projections are defined (will check and refine as necessary in step #4). Only the characteristics affecting distortion need to be specified at this point. Other parameters, such as false northings and eastings, will be specified after a design is selected based on distortion performance.

Projection:	**Transverse Mercator**	**Lambert Conformal Conic**
Projection axis:	λ_0 = 121° 15' 00" W	φ_0 = 44° 20' 00" N
Projection axis scale:	k_0 = 1.00013	k_0 = 1.00013

4. Compute distortion throughout project area and refine design parameters

The best approach for doing this is to compute the distortion on a regular high-resolution grid of points, so that distortion can be visualized and analyzed everywhere. That approach is used as part of the workshop that accompanies this document.

In addition to plotting a map of distortion, it can also be computed at discrete points. These points can be NGS control points, other surveyed points, or any point with a reasonable accurate topographic ellipsoid height. For this design example, the heights at the given location for each of the seven towns are used, which are accurate to about ±10 ft (corresponding to distortion accuracy of ±0.5 ppm). A computation example for each of the two initial LDP designs is provided for the point representing the town of Bend using the values from Table 5:

Bend: φ = 44° 03' 29" N, λ = 121° 18' 55" W, h = 3557 ft,

where linear distortion is computed as $\delta = k\left(\frac{R_G}{R_G + h}\right) - 1$

and geometric mean radius of curvature as $R_G = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$ = 20,923,218 ift for Bend.

The value of *k* can be computed using various geospatial software packages. If such software is not available, it can be computed using the equations given after this design example. The values obtained for the TM is *k* = 1.000 130 336, and for the LCC is *k* = 1.000 141 485.

Using these values gives the following values of distortion at the point in Bend:

TM: $\delta = 1.000130336 \times \left(\frac{20{,}923{,}218}{20{,}923{,}218 + 3557}\right) - 1$ = 0.999 960 341 – 1 = **–39.7 ppm**

LCC: $\delta = 1.000141485 \times \left(\frac{20{,}923{,}218}{20{,}923{,}218 + 3557}\right) - 1$ = 0.999 971 488 – 1 **= –28.5 ppm**

Despite using the mean topographic height of the seven towns for determining the projection scale, the distortion magnitude for both projections exceeds not only the ±10 ppm criterion for Bend, but even the overall target of ±20 ppm. This could be fixed for the point in Bend by increasing the projection scale by, say, 30 ppm to k_0 = 1.00016, which would change the values to –9.7 ppm and +1.5 ppm for the TM and LCC projections, respectively. However, this would also increase the distortion at the other points by 30 ppm, yielding a maximum in Madras of +57.3 ppm and +69.9 ppm for the TM and LCC projections, respectively. Such distortion is clearly much too large, so a different approach is needed.

Since simply changing the projection scale alone will not achieve the desired result, we can instead change the location of the projection axes. Changing their locations will change the variability of the distortion in the design area. We can assess the variability by the distortion range and standard deviation. The results of doing that for the TM and LCC projections are shown in tables 6 and 7, respectively. In addition to changing the projection axis locations, in all design alternatives the axis scale was also changed so that the mean distortion was within ±10 ppm.

Table 6. Distortion performance for six different TM projection alternatives (initial design is highlighted yellow).

TM axis scale	1.00012	1.00013	1.00013	1.00013	1.00013	1.00012
TM axis longitude	120° 45' W	121° 00' W	121° 10' W	121° 15' W	121° 20' W	121° 30' W
Location	**Linear distortion for TM projection (parts per million)**					
Bend	-24.8	-32.2	-38.3	-39.7	-40.0	-47.3
Redmond	-6.0	-7.7	-10.1	-9.6	-8.1	-11.7
Prineville	-13.9	-2.3	4.2	9.1	15.0	20.2
Madras	27.3	27.5	26.3	27.3	29.4	26.8
Sisters	21.1	4.7	-7.5	-11.9	-15.3	-28.7
Culver	14.3	11.1	7.8	7.7	8.7	4.0
Metolius	16.3	14.6	12.1	12.5	14.0	10.1
Mean	**4.9**	**2.2**	**-0.8**	**-0.7**	**0.5**	**-3.8**
Range	**52.1**	**59.6**	**64.6**	**67.0**	**69.4**	**74.1**
Std deviation	**±19.7**	**±19.0**	**±20.5**	**±21.8**	**±23.3**	**±26.9**

Table 7. Distortion performance for six different LCC projection alternatives (initial design is highlighted yellow; final design is highlighted green).

LCC axis scale	1.00013	1.00013	1.00012	1.00012	1.00011	1.00010
LCC axis latitude	40° 20' N	40° 30' N	40° 35' N	40° 40' N	40° 45' N	40° 50' N
Location	**Linear distortion for LCC projection (parts per million)**					
Bend	-28.5	-10.4	-8.2	6.0	12.4	20.8
Redmond	-9.5	-2.2	-5.4	3.5	4.4	7.5
Prineville	-4.3	1.6	-2.2	6.0	6.3	8.7
Madras	39.9	28.9	16.6	16.4	8.3	2.3
Sisters	-18.6	-12.3	-16.0	-7.5	-7.0	-4.4
Culver	13.2	7.7	-1.9	0.6	-4.8	-8.0
Metolius	21.8	13.2	2.1	3.1	-3.8	-8.7
Mean	**2.0**	**3.8**	**-2.2**	**4.0**	**2.3**	**2.6**
Range	**68.4**	**41.2**	**32.5**	**23.9**	**19.4**	**29.5**
Std deviation	**±24.1**	**±14.4**	**±10.1**	**±7.1**	**±7.4**	**±10.7**

As shown in Table 6, the distortion standard deviation and range of the TM design alternatives both change as the projection axis (central meridian) location is changed. However, the changes are generally modest, with no substantial improvement from the initial design.

In contrast, Table 7 shows that the change in distortion standard deviation and range of the LCC design alternatives is significant as the projection axis (central parallel) location is changed. The standard deviation and range decrease from ±24.1 and 68.4 from the initial design to a minimum of ±7.1 ppm (for φ_0 = 44° 40' N) and 19.4 ppm (for φ_0 = 44° 45' N).

The standard deviation is minimum for φ_0 = 44° 40' N, but the range is minimum for φ_0 = 44° 45' N. For the latter design, the distortion was becoming excessive in the southern end of the design region, as exemplified by the distortion of +12.4 ppm in Bend. For this case, the central parallel is far enough north that distortion in the southern part of the design area was changing too rapidly with change in latitude. Because of this affect, as well as inspection of performance in other areas of the design region (as shown on distortion maps), a design with φ_0 = 44° 40' N and k_0 = 1.00012 was selected for the final design (highlighted green in Table 7). The map in Figure 4 shows the distortion of this final design.

5. Keep the definition SIMPLE and CLEAN!

The LCC projection parameters affecting distortion were defined in the previous step and are given again in this step, along with the other needed parameters that do NOT affect distortion.

- LCC k_o defined to *exactly* FIVE decimal places: $\boldsymbol{k_0}$ **= 1.00012 (exact)**
- Both central parallel and central meridian are defined to nearest whole arc-minute. .

  $\boldsymbol{\varphi_0}$ **= 44° 40' 00" N = 31.666666666667°** and $\boldsymbol{\lambda_0}$ **= 121° 15' 00" W = −121.25°**

  The central meridian (λ_0) was selected as a clean value near the east-west center of the design area (has no affect on distortion).

  For an LCC projection, the latitude of grid origin must also be specified; it is the latitude where the false northing is defined (i.e., the northing on the central meridian at that latitude). It also has no affect on distortion, and it was set to equal to the central parallel. This was done to accommodate commercial software (such as Trimble) where the two must be the same.

- Grid origin is defined using clean whole values with as few digits as possible:

  $\boldsymbol{N_0}$ **= 130,000.000 m** and $\boldsymbol{E_0}$ **= 80,000.000 m**

  Metric values were used to avoid confusion between international and US survey feet in the defined parameters (as also done in Oregon State Plane). These values were selected to keep grid coordinates positive but as small as possible throughout the design area (and also distinct from State Plane and UTM coordinates).

6. *Explicitly* define linear unit and geometric reference system (i.e., geodetic datum)

- Linear unit is the **meter**, and geometric reference system (geodetic datum) is **NAD 83**
  - Although the projection parameters are defined in meters, the output coordinates are typically provided in international feet, as is done for Oregon State Plane.
  - Note that the geometric reference system definition is NAD 83 without a realization ("datum tag") specified such as "20110", per the previous discussion on LDP design. Exactly the same approach is used for State Plane; it is always referenced to "generic" NAD 83. Only the coordinates themselves are reference to a specific realization. But that has no affect on the projection or ellipsoid parameters.
- The projection parameters, linear unit, and geodetic datum can be used directly to create a coordinate system definition that is compatible with most surveying, engineering, GIS, and other geospatial software. For example, this can be done for Esri software by creating a projection file (*.prj), or for Trimble software by using Coordinate System Manager to augment the coordinate system database file (*.csd).
- The final design projection parameters are shown in Table 8, which are the values adopted for this as the Bend-Redmond-Prineville Zone of the Oregon Coordinate Reference System (OCRS). See the OCRS web page at www.oregon.gov/ODOT/HWY/GEOMETRONICS/Pages/ocrs.aspx for more information.
- Table 8 also includes projection parameters for State Plane Coordinate System of 1983, Oregon South Zone (SPCS 83 OR S), both as defined and "modified" to get "ground coordinates" in Bend. SPCS 83 OR S was scaled "to ground" by applying a scale factor of 1.000 160 760 so that distortion in Bend is the same as that of the OCRS. The modified SPCS 83 projection parameters were calculated from the scale factor, resulting in a "messy" definition (i.e., the false easting and scale have trailing digits after the decimal). This can make such systems problematic to use in modern geospatial software via formal projection definitions. For modified SPCS, the geodetic parameters (latitude and longitude) are unaffected; the central parallel latitude and scale for a two-parallel LCC is a computed quantity (implicitly defined by the two standard parallels).
- The performance of "modified" State Plane systems is usually inferior to a carefully designed LDP. Table 9 gives the difference in performance for the seven locations between the OCRS LDP designed in this example and SPCS 83 OR S, both the original definition and the modified one (defined in Table 8). For the modified SPCS 83 system, note that although the distortion in Bend is the same (6.0 ppm), the distortion elsewhere is much greater, with a mean of +152 ppm (versus +4 ppm for the OCRS zone). Note also that both the original and modified version have essentially the same distortion range and standard deviation (274 and ±97 ppm, respectively). Compare this to the much lower variability of the OCRS zone, with a range of 24 ppm and standard deviation of ±7 ppm. Figure 5 is a distortion map for this modified SPCS 83 OR S system, with the same distortion color ramp as the OCRS Bend-Redmond-Prineville Zone map in Figure 4. The difference in distortion distribution between the two figures is striking.

Table 8. Comparison of OCRS Bend-Redmond-Prineville Zone parameters to State Plane Coordinate System of 1983, Oregon South Zone (SPCS 83 OR S) and "equivalent" back-calculated modified SPCS 83 OR S scaled to match OCRS distortion in Bend.

Transverse Mercator projection parameters	OCRS Bend-Redmond-Prineville Zone	SPCS 83 Oregon South Zone	"Modified" SPCS 83 Oregon South Zone (for Bend)
Central standard parallel	44° 40' 00" N	43° 10' 06.91956…" N (computed)	43° 10' 06.91956…" N (computed)
North standard parallel	n/a	44° 00' 00" N	44° 00' 00" N
South standard parallel	n/a	42° 20' 00" N	42° 20' 00" N
Latitude of grid origin	44° 40' 00" N	41° 40' 00" N	41° 40' 00" N
Central meridian longitude	121° 15' 00" W	120° 30' 00" W	120° 30' 00" W
False northing	130,000 m (exact)	0 m (exact)	0 m (exact)
False easting	80,000 m (exact)	1,500,000 m (exact)	1,500,241.14 m
Central parallel scale factor	1.00012 (exact)	0.999 894 607 592 09… (computed)	1.000 055 350 649 21… (computed*)

*Computed by applying 1.000 160 760 scale factor to original SPCS 83 central parallel scale.

Table 9. Comparison of distortion for OCRS Bend-Redmond-Prineville zone, SPCS 83 OR S, and modified SPCS 83 OR S (modified such that distortion is same as OCRS in Bend).

Location	Linear distortion (parts per million)		
	OCRS Bend-Redmond-Prineville Zone	SPCS 83 Oregon South Zone	"Modified" SPCS 83 Oregon South Zone
Bend	6.0	-154.7	6.0
Redmond	3.5	-59.4	101.4
Prineville	6.0	-44.4	116.3
Madras	16.4	119.1	279.9
Sisters	-7.5	-62.0	98.8
Culver	0.6	53.8	214.6
Metolius	3.1	84.3	245.0
Mean	**4.0**	**-9.0**	**151.7**
Range	**23.9**	**273.8**	**273.9**
Std deviation	**±7.1**	**±97.4**	**±97.4**

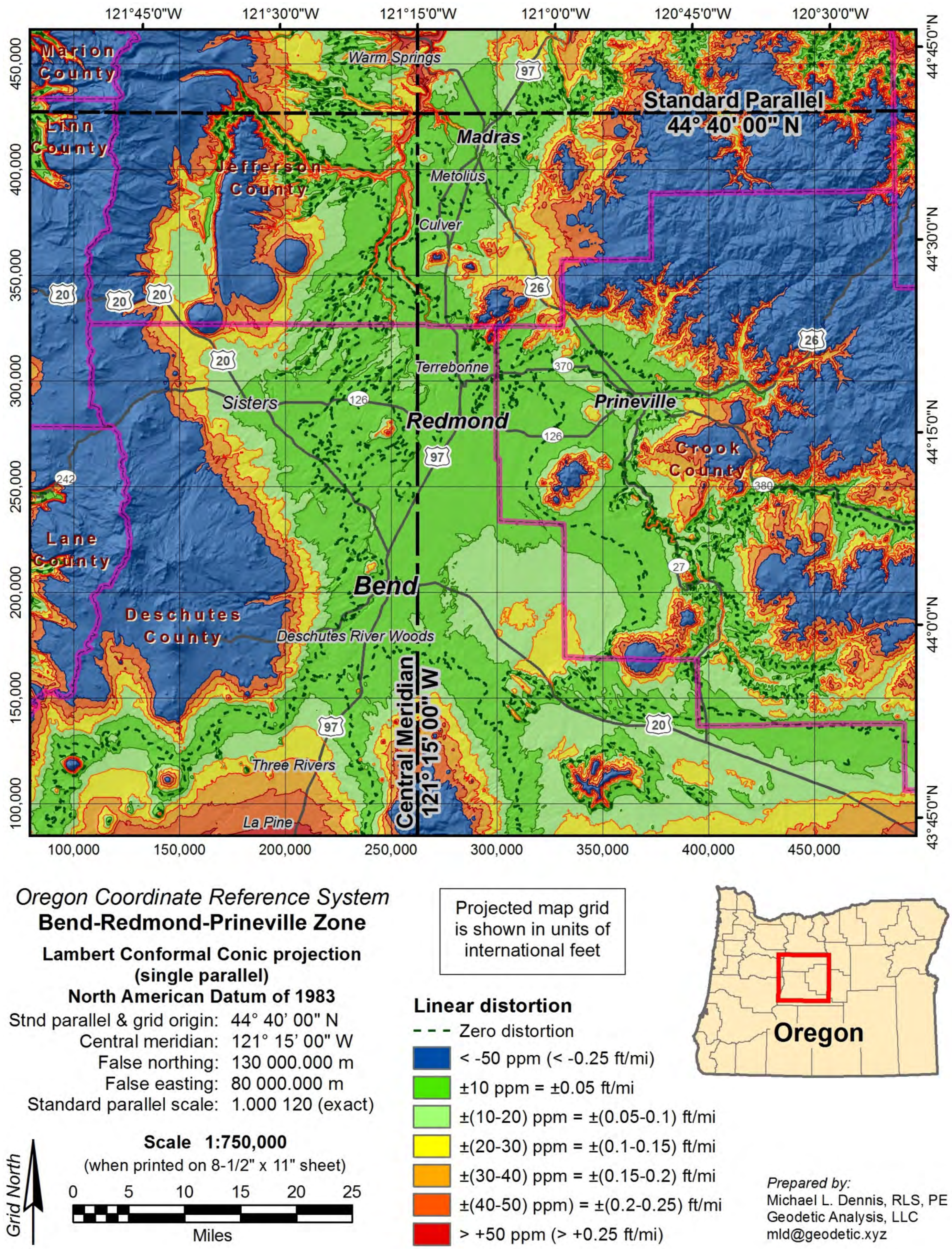

Figure 4. Linear distortion for OCRS Bend-Redmond-Prineville Zone.

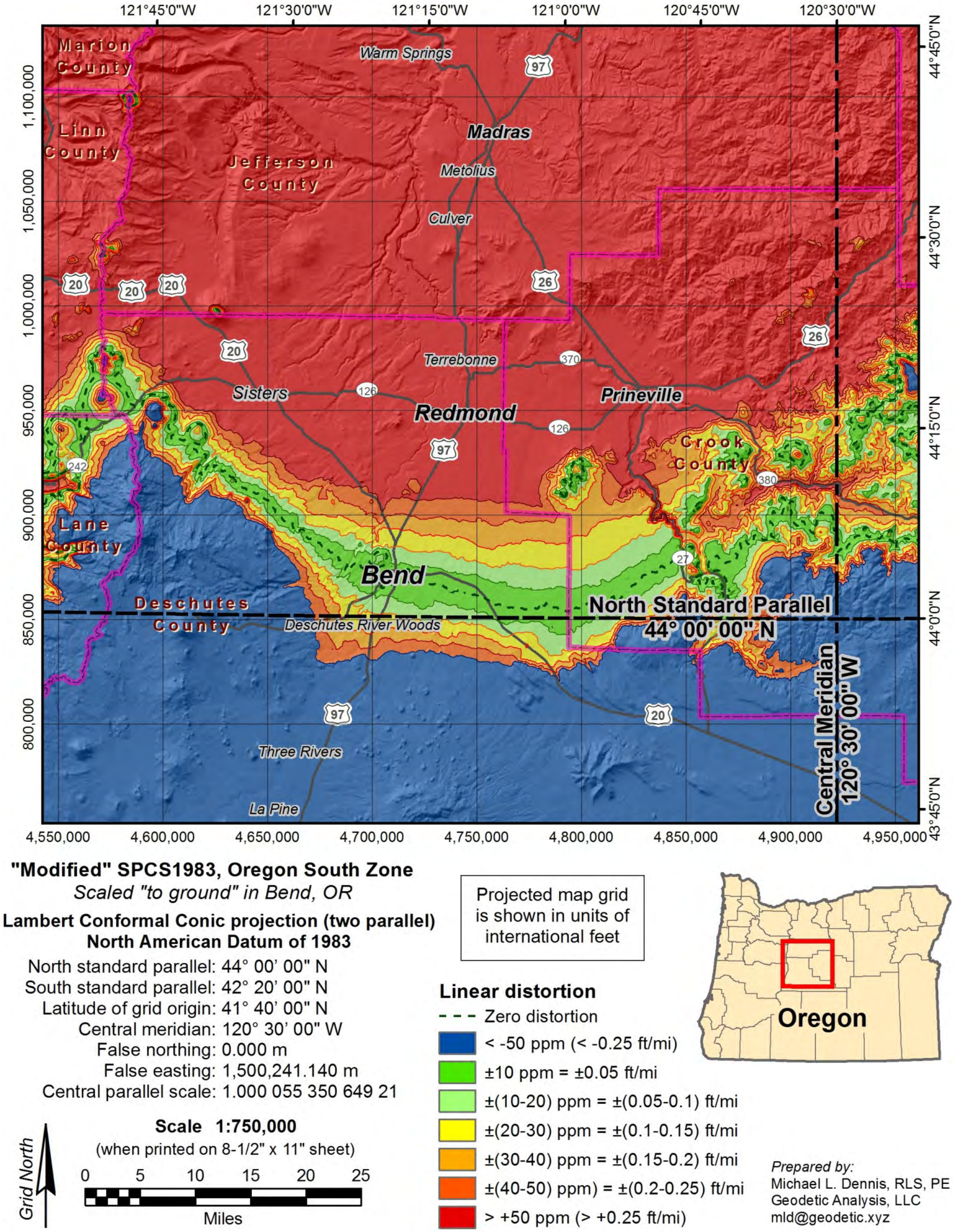

Figure 5. Distortion for "modified" SPCS 83 OR S scaled "to ground" in Bend (compare to Figure 4).

Projection grid point scale factor and convergence angle computation

The projection grid point scale factor, *k*, is required to compute map projection distortion for a point on the ground. Because some surveying, engineering, and GIS software does not provide *k*, formulas for computing it are given below for the Transverse Mercator and Lambert Conformal Conic projections. These equations were derived from those provided in *NOAA Manual NOS NGS 5* "State Plane Coordinate System of 1983" by James Stem (1990). Equations for computing the convergence angle of these projections are also provided.

For the ***Transverse Mercator*** (TM) projection, the grid scale factor at a point can be computed as follows (derived from Stem, 1990, pp. 32-35):

$$k = k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1-e^2} \right) \left[1 + \frac{(\Delta\lambda \cos\varphi)^2}{12} \left(5 - 4\tan^2\varphi + \frac{e^2\cos^2\varphi}{1-e^2} \left(9 - 24\tan^2\varphi \right) \right) \right] \right\}$$

where $\Delta\lambda = \lambda_0 - \lambda$ (in radians; note that west longitude is negative)

λ = geodetic longitude of point

λ_0 = central meridian longitude

and all other variables are as defined previously.

The following shorter equation can be used to approximate *k* for the TM projection. It is accurate to better than 0.02 part per million (at least 7 decimal places) if the computation point is within about ±1° of the central meridian (about 80-100 km or 50-60 miles between latitudes of 30° and 45°).

$$k \approx k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1-e^2} \right) \right\}$$

Note that this equation may not be sufficiently accurate for computing *k* throughout a UTM system zone (at the zone width of ±3° from the central meridian the error can exceed 1 ppm).

An even simpler equation can be used to approximate the grid scale factor, which utilizes the grid coordinate easting value and is about twice as accurate as the previous equation (i.e., better than 0.01 part per million if the computation point is within about ±1° of the central meridian):

$$k \approx k_0 + \frac{(E_0 - E)^2}{2(k_0 R_G)^2}$$

where E = Easting of the point where *k* is computed (in same units as R_G)

E_0 = False easting (on central meridian) of projection definition (in same units as R_G)

R_G = Earth geometric mean radius of curvature (can estimate using 6,373,000 meters or 20,910,000 feet for coterminous US)

For the ***Lambert Conformal Conic*** (LCC) projection, the grid scale factor at a point can be computed as follows (derived from Stem, 1990, pp. 26-29):

$$k = k_0 \frac{\cos\varphi_0}{\cos\varphi}\sqrt{\frac{1-e^2\sin^2\varphi}{1-e^2\sin^2\varphi_0}}\ \exp\left\{\frac{\sin\varphi_0}{2}\left[\ln\frac{1+\sin\varphi_0}{1-\sin\varphi_0} - \ln\frac{1+\sin\varphi}{1-\sin\varphi} + e\left(\ln\frac{1+e\sin\varphi}{1-e\sin\varphi} - \ln\frac{1+e\sin\varphi_0}{1-e\sin\varphi_0}\right)\right]\right\}$$

where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if k_0 = 1)

φ_0 = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

and all other variables are as defined previously. In order to use this equation for a two-parallel LCC, the two-parallel LCC must first be converted to an equivalent one-parallel LCC by computing φ_0 and k_0. The equations to do this are long, but are provided here for the sake of completeness. For a two-parallel LCC, the central parallel is

$$\varphi_0 = \sin^{-1}\left[\frac{2\ln\left(\dfrac{\cos\varphi_S}{\cos\varphi_N}\sqrt{\dfrac{1-e^2\sin^2\varphi_N}{1-e^2\sin^2\varphi_S}}\right)}{\ln\left(\dfrac{1+\sin\varphi_N}{1-\sin\varphi_N}\right) - \ln\left(\dfrac{1+\sin\varphi_S}{1-\sin\varphi_S}\right) + e\left[\ln\left(\dfrac{1+e\sin\varphi_S}{1-e\sin\varphi_S}\right) - \ln\left(\dfrac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right)\right]}\right],$$

and the central parallel scale factor is

$$k_0 = \frac{\cos\varphi_N}{\cos\varphi_0}\sqrt{\frac{1-e^2\sin^2\varphi_0}{1-e^2\sin^2\varphi_N}}$$

$$\times\ \exp\left\{\frac{\sin\varphi_0}{2}\left[\ln\left(\frac{1+\sin\varphi_N}{1-\sin\varphi_N}\right) - \ln\left(\frac{1+\sin\varphi_0}{1-\sin\varphi_0}\right) + e\left(\ln\left[\frac{1+e\sin\varphi_0}{1-e\sin\varphi_0}\right] - \ln\left[\frac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right]\right)\right]\right\},$$

where φ_N and φ_S = geodetic latitude of northern and southern standard parallels, respectively, and all other variables are as defined previously.

Convergence angles. For the TM, the convergence angle can be approximated as $\gamma = -\Delta\lambda\sin\varphi$ (where all variables are as defined previously; the units of γ are the same as the units of $\Delta\lambda$). This equation is accurate to better than ±0.2 arc-second if the computation point is within about ±1° of the central meridian. For any LCC, the convergence angle is exactly equal to $\gamma = -\Delta\lambda\sin\varphi_0$.

Surveying & mapping spatial data requirements & recommendations

These should be explicitly specified in surveying and mapping projects

1. Completely define the coordinate system

a. Linear unit (e.g., international foot, U.S. survey foot, meter)
   i. Use same linear unit for horizontal and vertical coordinates
b. Geodetic datum (recommend North American Datum of 1983)
   i. Should include "datum tag", e.g., 1986, 1991, 1998, 2007, 2011, as necessary, as well as epoch date for modern high-accuracy positions, e.g., 2010.00
   ii. WGS 84, ITRF/IGS, and NAD 27 are **NOT** recommended
c. Vertical datum (e.g., North American Vertical Datum of 1988)
   i. If GPS used for elevations, recommend using a modern geoid model (e.g., GEOID12A)
   ii. Recommend using NAVD 88 rather than NGVD 29 when possible
d. Map projection type and parameters (e.g., Transverse Mercator, Lambert Conformal Conic)
   i. Special attention required for low-distortion grid (a.k.a. "ground") coordinate systems
      1) Avoid scaling of existing coordinate systems (e.g., "modified" State Plane)

2. Require *direct* referencing of the NSRS (National Spatial Reference System)

a. Ties to published control strongly recommended (e.g., National Geodetic Survey control)
   i. Relevant component of control must have greater accuracy than positioning method used
      1) E.g., network accuracies that meet project needs, 2nd order (or better) for vertical control
b. NGS Continuously Operating Reference Stations (CORS) can be used to reference the NSRS
   i. Free Internet GPS post-processing service: OPUS (Online Positioning User Service)

3. Specify *accuracy* requirements (*not* precision)

a. Use objective, defensible, and robust methods (published ones are recommended)
   i. Mapping and surveying: National Standard for Spatial Data Accuracy (NSSDA)
      1) Require occupations ("check shots") of known high-quality control stations
   ii. Surveys performed for establishing control or determining property boundaries:
      1) Appropriately constrained and over-determined least-squares adjusted control network
      2) Beware of "cheating" (e.g., using "trivial" GPS vectors in network adjustment)

4. Documentation is *essential* (metadata!)

a. Require a report detailing methods, procedures, and results for developing final deliverables
   i. This must include any and all post-survey coordinate transformations
      1) E.g., published datum transformations, computed correction surfaces, "rubber sheeting"
b. Documentation should be complete enough that someone else can reproduce the product
c. For GIS data, recommend that accuracy and coordinate system information be included as feature attributes (not just as separate, easy-to-lose and easy-to-ignore metadata files)

Example of surveying and mapping documentation (*metadata*)

Basis of Bearings and Coordinates

Linear unit: International foot (ift)

Geodetic datum: North American Datum of 1983 (2011) epoch 2010.00

Vertical datum: North American Vertical Datum of 1988 (see below)

System: Oregon Coordinate Reference System

Zone: Bend-Redmond-Prineville

Projection: Lambert Conformal Conic (one-parallel)
Standard parallel and latitude of grid origin: 44° 40’ 00” N
Longitude of central meridian: 121° 15’ 00” W
Northing at grid origin: 130,000.000 m (~426,509.18635 ift)
Easting at central meridian: 80,000.000 m (~262,467.19160 ift)
Scale factor on central meridian: 1.000 12 (exact)

All distances and bearings shown hereon are projected (grid) values based on the preceding projection definition. The projection was defined to minimize the difference between projected (grid) distances and horizontal (“ground”) distances at the topographic surface within the design area of this coordinate system.

The basis of bearings is geodetic north. Note that the grid bearings shown hereon (or implied by grid coordinates) do not equal geodetic bearings due to meridian convergence.

Orthometric heights (elevations) were transferred to the site from NGS control station “C 30” (PID QD0823) using GNSS with NGS geoid model “GEOID12A” referenced to the current published 1st order NAVD 88 height of this station (1049.170 m).

The survey was conducted using GNSS referenced to the National Spatial Reference System. A partial list of point coordinates is given below (additional coordinates are available upon request). Accuracy estimates are at the 95% confidence level and are based on an appropriately constrained and weighted least-squares adjustment of redundant observations.

Point #1, NGS control station C 30 (PID QD0823), constrained (off site)		**Estimated accuracy**
Latitude = 44° 06' 53.98076'' N	Northing = 225,363.515 ift	Horiz = ±0.024 ift
Longitude = 121° 17' 27.31006'' W	Easting = 251,718.529 ift	Ellipsoid ht = ±0.076 ift
Ellipsoid height = 3372.940 ift	Elevation = 3442.159 ift	Elevation FIXED

Point #1002, 1/2” rebar with aluminum cap, derived coordinates		**Estimated accuracy**
Latitude = 44° 06' 31.96763'' N	Northing = 223,132.860 ift	Horiz = ±0.034 ift
Longitude = 121° 16' 51.33054'' W	Easting = 254,342.973 ift	Ellipsoid ht = ±0.086 ift
Ellipsoid height = 3395.610 ift	Elevation =3464.760 ift	Elevation = ±0.094 ift

Point #1006, 1/2” rebar with plastic cap, derived coordinates		**Estimated accuracy**
Latitude = 44° 06' 28.79196'' N	Northing = 222,811.061 ift	Horiz = ±0.047 ift
Longitude = 121° 16' 45.17852'' W	Easting = 254,791.795 ift	Ellipsoid ht = ±0.088 ift
Ellipsoid height = 3391.047 ift	Elevation = 3460.184 ift	Elevation = ±0.097 ift

Selected References

National Geodetic Survey (www.ngs.noaa.gov) web pages of particular interest

Control station datasheets: www.ngs.noaa.gov/cgi-bin/datasheet.prl
The Geodetic Tool Kit: www.ngs.noaa.gov/TOOLS/
Online Positioning User Service (OPUS): www.ngs.noaa.gov/OPUS/
Continuously Operating Reference Stations (CORS): www.ngs.noaa.gov/CORS/
The Geoid Page: www.ngs.noaa.gov/GEOID/
NGS State Geodetic Advisors: www.ngs.noaa.gov/ADVISORS/AdvisorsIndex.shtml

Other documents on projections, datums, and geospatial metadata

Armstrong, M.L., Singh, R., and Dennis, M.L., 2014. *Oregon Coordinate Reference System Handbook and User Guide*, version 2.01, Oregon Department of Transportation, Geometronics Unit, Salem, Oregon, USA, 65 pp., ftp.odot.state.or.us/ORGN/Documents/ocrs_handbook_user_guide.pdf.

Dennis, M.L., Miller, N., and Brown, G., 2014. *Iowa Regional Coordinate System Handbook and User Guide*, version 2.10, Iowa Department of Transportation, 76 pp., www.iowadot.gov/rtn/pdfs/IaRCS_Handbook.pdf.

Federal Geographic Data Committee, 1998. *Geospatial Positioning Accuracy Standards*, FGDC-STD-007.2-1998, Federal Geographic Data Committee, Reston, Virginia, USA, 128 pp., www.fgdc.gov/standards/projects/FGDC-standards-projects/accuracy/, [includes Reporting Methodology (Part 1), Standards for Geodetic Networks (Part 2), National Standard for Spatial Data Accuracy (Part 3), Standards for Architecture, Engineering, Construction (A/E/C) and Facility Management (Part 4), and Standards for Nautical Charting Hydrographic Surveys (Part 5)].

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National Geospatial-Intelligence Agency, 2014a. *Department of Defense World Geodetic System of 1984: Its Definition and Relationships with Local Geodetic Systems*, version 1.0.0, NGA.STND.0036_1.0.0_WGS8 (National Geospatial-Intelligence Agency Standardization Document), 2017 pp., earth-info.nga.mil/GandG/publications/NGA_STND_0036_1_0_0_WGS84/NGA.STND.0036_1.0.0_WGS84.pdf.

National Geospatial-Intelligence Agency, 2014b. *The Universal Grids and the Transverse Mercator and Polar Stereographic Map Projections*, version 2.0.0, NGA.SIG.0012_2.0.0_UTMUPS (National Geospatial-Intelligence Agency Standardization Document), 86 pp., earth-info.nga.mil/GandG/publications/NGA_SIG_0012_2_0_0_UTMUPS/NGA.SIG.0012_2.0.0_UTMUPS.pdf.

National Geospatial-Intelligence Agency, 2014c. *Universal Grids and Grid Reference Systems*, version 2.0.0, NGA.STND.0037_2.0.0_GRIDS (National Geospatial-Intelligence Agency Standardization Document), 101 pp., earth-info.nga.mil/GandG/publications/NGA_STND_0037_2_0_0_GRIDS/NGA.STND.0037_2.0.0_GRIDS.pdf.

Snyder, J.P., 1987. *Map Projections — A Working Manual*, U.S. Geological Survey Professional Paper 1395, Washington, D.C., USA, 383 pp, pubs.er.usgs.gov/djvu/PP/PP_1395.pdf.

Stem, J.E., 1990. State Plane Coordinate System of 1983, *NOAA Manual NOS NGS 5*, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Geodetic Survey, Rockville, Maryland, USA, 119 pp., www.ngs.noaa.gov/PUBS_LIB/ManualNOSNGS5.pdf.

Van Sickle, J., 2004. *Basic GIS Coordinates*, CRC Press LLC, Boca Raton, Florida, USA, 173 pp.